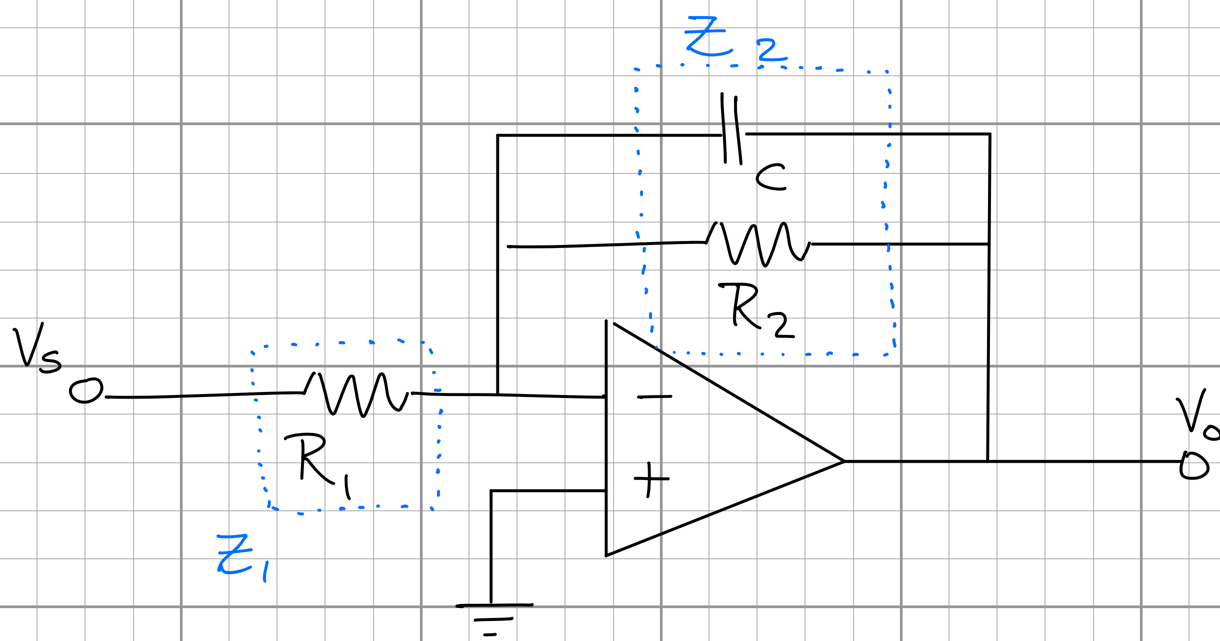
Transfer Function - First-order inverting LPF



Q: What is $H(j\omega)$?

1) Simplify using impedance representation.

- $Z_R = R$
- $Z_C = \frac{1}{j\omega C} = -j\left(\frac{1}{\omega C}\right)$

2) Use op-amp ideal assumptions to solve for v_o/v_s.

(i) $V_- = V_+$

(ii) $i_- = i_+ = 0$

1) $Z_1 = R_1$, $Z_{R_2} = R_2$, $Z_C = \frac{1}{j\omega C}$

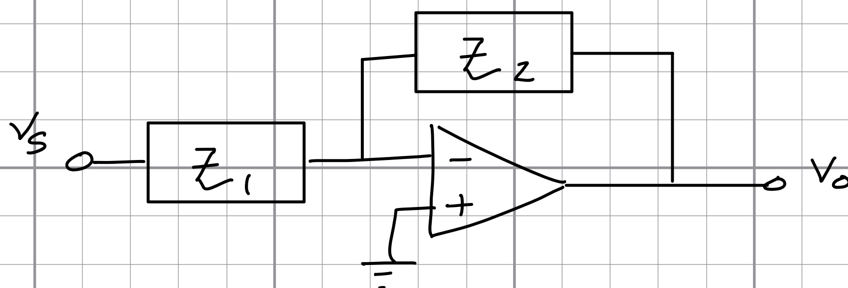
$$\frac{1}{Z_2} = \frac{1}{Z_{R_2}} + \frac{1}{Z_C} \longrightarrow Z_{R_2} Z_C = (Z_C + Z_{R_2}) Z_2$$

$$\therefore Z_2 = Z_{R_2} Z_C / (Z_C + Z_{R_2})$$

$$Z_2 = \frac{R_2\left(\frac{1}{j\omega C}\right)}{R_2 + \frac{1}{j\omega C}} \quad \frac{(j\omega C)}{(j\omega C)}$$

$$\boxed{= \frac{R_2}{j\omega R_2 C + 1}}$$

we aim to simplify to: $\frac{a+jb}{c+jd}$



2)

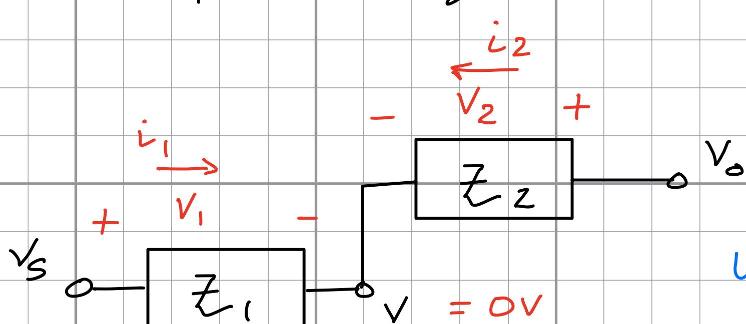
(i) compute voltage at inputs.

(ii) compute current along path.

(i) Note V_+ is held at ground.

$\therefore \quad V_+ = 0V = V_-$

(ii) Set-up KCL using other branch.



KCL

$i_1 + i_2 = 0$

$i_1 = -i_2$

w.r.t. new variables

$V_o = V_2 = i_2 Z_2$

$$i_1 = \frac{V_s - V_o}{Z_1} = \frac{V_s}{Z_1}$$

$$i_2 = \frac{V_o - V_-}{Z_2} = \frac{V_o}{Z_2}$$

Sub into KCL expression

$$\frac{V_s}{Z_1} = -\frac{V_o}{Z_2}$$

then solve for V_o

$$V_o = -\frac{Z_2}{Z_1} V_s$$

Sub. in expressions for Z_1 & Z_2.

$$\frac{V_o}{V_s} = -Z_2\left(\frac{1}{Z_1}\right)$$

$$= -\frac{R_2}{1 + j\omega R_2 C}\left(\frac{1}{R_1}\right)$$

$$\boxed{\frac{V_o}{V_s} = H(j\omega) = -\frac{R_2}{R_1}\left(\frac{1}{1 + j\omega R_2 C}\right)}$$

What is the cutoff frequency?

↳ observe pole location

$$1 + j\omega R_2 C = 0 \longrightarrow j\omega = -\frac{1}{R_2 C}$$

compute magnitude & solve for f_c $(\omega = 2\pi f_c)$

$$|j\omega| = \sqrt{0^2 + \omega^2} = \omega$$

$$\left| -\frac{1}{R_2 C} \right| = \sqrt{\left(-\frac{1}{R_2 C}\right)^2 + 0^2} = \frac{1}{R_2 C}$$

$$\omega_c = 2\pi f_c = \frac{1}{R_2 C}$$

$$\boxed{\therefore \; f_c = \frac{1}{2\pi R_2 C}}$$